[This question paper contains 8 printed pages.]

(14) **Your Roll No.** 2023

Sr. No. of Question Paper : 5025 E

Unique Paper Code : 62354443

Name of the Paper : Analysis (LOCF)

Name of the Course : **B.A. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

4. **All** questions carry equal marks.

1. (a) Let S be a non empty bounded set in $\mathbb{R}$. Let $a > 0$, and let $aS = \{as : s \in S\}$. Prove that $\inf aS = a \inf S$, $\sup aS = a \sup S$.

P.T.O.

(b) Define order completeness property of real numbers.

(c) Define limit point of a set. Show that the set ℕ of natural numbers has no limit point.

(d) State and prove Archimedean property of real numbers.

2. (a) Show that the function defined as

$$f(x)=\begin{cases} x, & \text{if } x \text{ is irrational} \\ -x, & \text{if } x \text{ is rational} \end{cases}$$

is continuous only at $x = 0$.

(b) Show that the function f defined by $f(x) = x^2$ is uniformly continuous on $[-2,2]$.

(c) Define an open set. Prove that every open interval is an open set. Which of the following sets are open.

(i) $]2, \infty[$

(ii) $[3,4[$

(d) Let A and B be bounded nonempty subsets of R, and let $A + B = \{a + b : a \in A, b \in B\}$. Prove that $\sup(A + B) = \sup A + \sup B$.

3\. (a) Prove that every convergent sequence is bounded. Justify by an example that the converse is not true.

(b) Prove that the sequence $\langle a_n \rangle$ defined by the recursion formula :

$$a_1 = \sqrt{7}, a_{n+1} = \sqrt{7 + a_n}$$

converges to the positive root of $x^2 - x - 7 = 0$.

(c) State Cauchy's convergence criterion for sequences. Check whether the sequence $\langle a_n \rangle$, where

$$a_n = 1 + \frac{1}{5} + \frac{1}{9} + \dots\dots\dots + \frac{1}{4n-3}$$

is convergent or not.

(d) Test for convergence the series :

$$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots\dots\dots\dots\dots\dots$$

4. (a) Prove that, if the series $\sum u_n$ converges, then $\lim_{n\to\infty} u_n = 0$. Show by an example that the converse is not true.

(b) Test for convergence the series :

$$\sum_{n=1}^{\infty} \frac{2.4.6 \ldots\ldots\ldots (2n+2)}{3.5.7 \ldots\ldots\ldots (2n+3)} x^{n-1} \qquad (x > 0)$$

(c) Let $\langle a_n \rangle$ be a sequence defined by :

$$a_1 = 1,\ a_{n+1} = \frac{3+2a_n}{2+a_n},\ n \geq 1.$$

Show that $\langle a_n \rangle$ is convergent and find its limit.

(d) Prove that a sequence of real numbers converges if and only if it is a Cauchy sequence.

P.T.O.

5. (a) State Leibnitz test for convergence of an alternating series of real numbers. Apply it to test for convergence the series

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

(b) Show the sequence defined by $\langle a_n \rangle = \langle n^2 \rangle$ is not a Cauchy sequence.

(c) Prove that the sequence $\langle a_n \rangle$ defined by the relation,

$$a_n = 1,\ a_n = 1+\frac{1}{1!}+\frac{1}{2!}+\cdots\cdots\cdots+\frac{1}{(n-1)!},\quad (n \geq 2),$$

converges.

(d) Prove that every continuous function is integrable.

6. (a) Define Riemann integrability of a bounded function f on a bounded closed interval [a, b]. Show that the function f defined on [a, b] as

$$f(x)=\begin{cases}0 & \text{when } x \text{ is rational}\\ 1 & \text{when } x \text{ is irrational}\end{cases}$$

is not Riemann integrable.

(b) Test for convergence the series

$$\sum_{n=1}^{\infty}(-1)^n \frac{\cos n\alpha}{\sqrt{n^3}},\ \alpha \text{ being real.}$$

(c) State D'Alembert's ratio test for the convergence of a positive term series. Use it to test for convergence the series $\sum_{n=1}^{\infty}\frac{n!}{n^n}$.

P.T.O.

(d) Show that $f(x) = \frac{1}{x}$ is not uniformly c

$[0,1]$.